HEAT TRANSFER IN THE VICINITY OF THE STAGNATION POINT IN AN AXISYMMETRIC JET FLOWING OVER FLAT SURFACES NORMAL TO THE FLOW

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The results of an analytical and experimental investigation of heat transfer in the vicinity of the stagnation point of a disk arranged normal to the flow and washed by an axisymmetric jet are presented.

In [1] an approximate analytical calculation was made for the laminar boundary layer between an axisymmetric jet and a plate arranged normal to the flow, but the stagnation point region was excluded from consideration. Hydrodynamic investigations of the boundary layer at the wall [2] have permitted calculation of the laminar boundary layer in the vicinity of the stagnation point, when the flow must be regarded as gradient flow.

In the case of an axisymmetric laminar boundary layer with a pressure gradient the momentum equation may be written in the following form [3]:

$$U_s \frac{dZ}{dr} = F(\varkappa) - \frac{2\varkappa}{r}\frac{U_s}{U_s'}, \tag{1}$$

where $\varkappa = ZU_s'$, $Z = \vartheta^2/\nu$, ϑ is the momentum thickness, and U_s' is the derivative of velocity U_s with respect to r.

In the region of the stagnation point (according to the data of [2], this region falls in the range $0 < \bar{r} \leq \bar{r}_m$), the variation of the relative axial velocity of the jet U_s, taken as the velocity at the outer edge of the boundary layer, over the radius of the disk, is determined by the equation

$$\bar{U}_s = a\bar{r} - b\bar{r}^3. \tag{2}$$

The parameters a and b depend on the relative distance of the nozzle from the plate $\bar{h}$ and are equal to

$$\text{when } \bar{h} \leqslant 6.2 \quad a = 1.5\bar{h}^{-0.22}, \quad b = 0.5\bar{h}^{-0.42};$$

$$\text{when } \bar{h} \geqslant 6.2 \quad a = 16.1\bar{h}^{-1.54}, \quad b = 47\bar{h}^{-2.94}. \tag{3}$$

The integration of (1) reduces to a quadrature, since the function $F(\varkappa)$ in the interval between the stagnation point ($\varkappa_0 = 0.057$, $\lambda_0 = 4.716$) and the point r_m ($\varkappa_m = 0$, $\lambda_m = 0$) may be replaced, with a good approximation, as shown in [4], by the straight line $F(\varkappa) = 0.47 - 8\varkappa$. Then (1) is rewritten as

$$\frac{dZ}{dr} = \frac{1}{U_s}\left(0.47 - 8\varkappa - \frac{2\varkappa}{r}\frac{U_s}{U_s'}\right),$$

or

$$\frac{dZ}{dr} + Z\left(\frac{2}{r} + 8\frac{U_s'}{U_s}\right) = \frac{0.47}{U_s}.$$

The solution is elementary:

$$Z = \frac{0.047\, d_0}{U_0 a\left(1 - k\bar{r}^2\right)^8} F(k\bar{r}^2),$$

where $k = b/a$ and

$$F(k\bar{r}^2) = 1 - \frac{35}{6}k\bar{r}^2 + 15k^2\bar{r}^4 - \frac{175}{8}k^3\bar{r}^6 + \frac{175}{9}k^4\bar{r}^8 - \frac{21}{2}k^5\bar{r}^{10} + \frac{35}{11}k^6\bar{r}^{12} - \frac{1}{24}k^7\bar{r}^{14}.$$

The quantity $k\bar{r}^2$ varies in the limits 0 to 0.33. The function $F(k\bar{r}^2)$ in the range of variation of $k\bar{r}^2$ indicated may be approximated by the expression $F(k\bar{r}^2) = (1 - k\bar{r}^2)^{5.6}$; then we finally obtain

$$Z = \frac{0.047\, d_0}{U_0 a\left(1 - k\bar{r}^2\right)^{2.4}}.$$

From the relation between Z and the momentum thickness ϑ ($Z = \vartheta^2/\nu$), we calculate the momentum thickness

$$\vartheta = \left[\frac{0.047\, d_0}{U_0 a\left(1 - k\bar{r}^2\right)^{2.4}}\right]^{1/2}.$$

According to [5], the thickness of the hydrodynamic boundary layer is determined by the equation

$$\vartheta/\delta = 37/315 - \lambda/945 - \lambda^2/9072.$$

Between the second shape factor $\varkappa$ and the first, λ, there is a universal relation [5]

$$\varkappa = (37/315 - \lambda/945 - \lambda^2/9072)^2\lambda,$$

which deviates only slightly, especially in the region of the stagnation point, from the line $\varkappa = 0.012\lambda$. Then

$$37/315 - \lambda/945 - \lambda^2/9072 = (0.012)^{1/2} \quad \text{and} \quad \vartheta/\delta = 0.1095.$$

From the last expression, the thickness of the hydrodynamic boundary layer is determined as

$$\delta = 1.97\, d_0/a^{1/2}\, \mathrm{Re}_0^{1/2}\left(1 - k\bar{r}^2\right)^{1.2}. \tag{4}$$

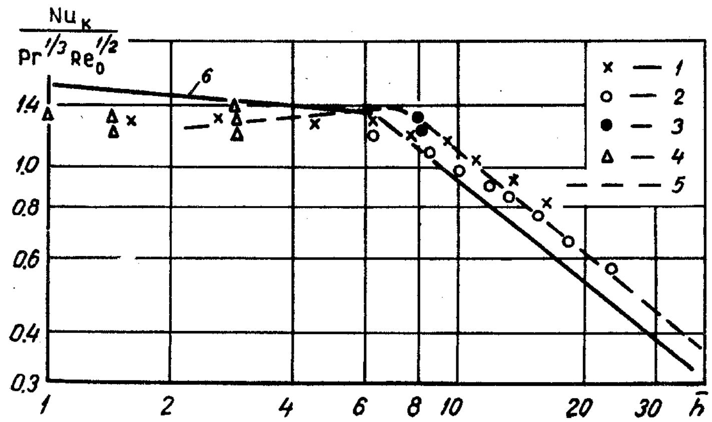

Fig. 1. Heat transfer coefficient at the stagnation point: 1) when d_0 = 40 mm and Re_0 = 31 000; 2) 31 and 40 000 (authors' data); 3) 16.5 and 10 000 to 30 000 (tests of [8]); 4) 178 and 40 000 to 550 000 (tests of [7]); 5) 6.35 and 7000; 28 000; 56 000 (tests of [6]); 6) according to Eqs. (9) and (10).

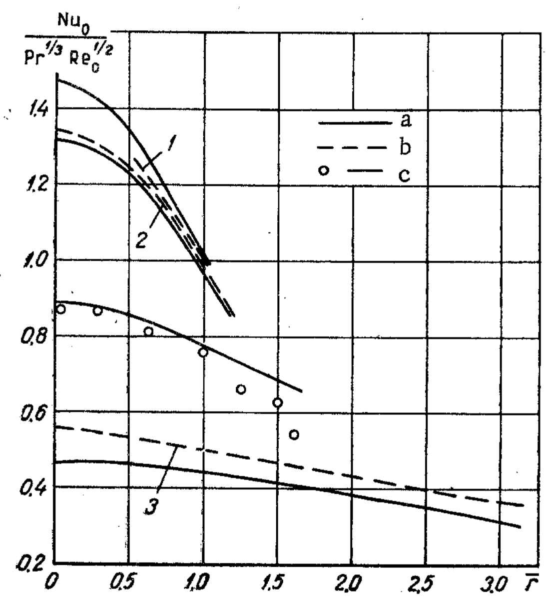

Fig. 2. Variation of local heat transfer coefficients over the disk radius in the gradient flow region ($0 \le \bar{r} \le \bar{r}_m$): a) calculations according to (7) and (8); b) test data of [6] for $\bar{h}$ = 2 (1), 6 (2), and 8 (3); c) authors' experiments with $\bar{h}$ = 10.

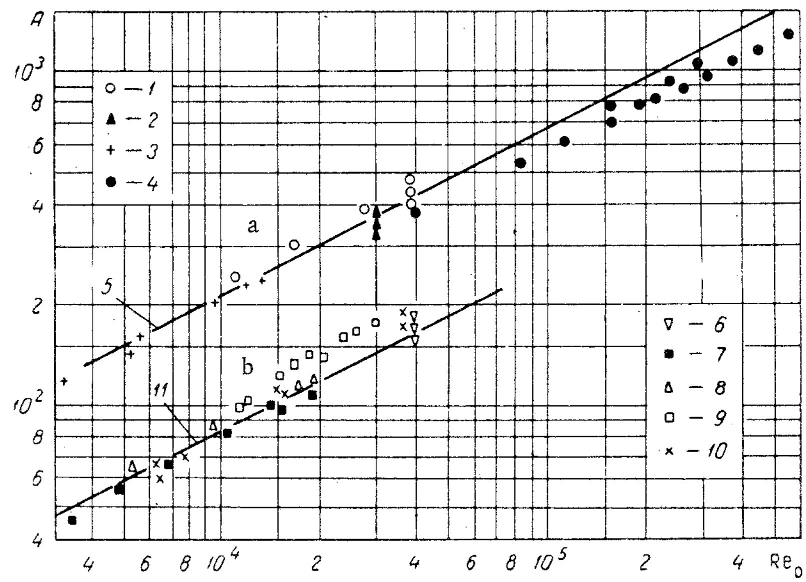

Fig. 3. Comparison of test data on mean heat transfer coefficient in the stagnation point region ($0 \le \bar{r} \le \bar{r}_m$) with theory: a) when $\bar{h} < 6.2$, $A = [\overline{Nu}_0\ \bar{R}^2] / \{Pr^{1/3}\bar{h}^{0.09} \times [1 - (1 - \bar{R}^2/3\bar{h}^{0.2})^{2.2}]\}^{-1}$ [1) when $d_0 = 31$ mm, $\bar{h} = 1.5–6.0$ and $\bar{R} = 0.83$; 2) 40; 1.5–6.0 and 0.62; 3) 20; 5 and 1.25, authors' experiments; 4) 178; 1.4–2.8 and 0.28, tests of [7]; 5) according to Eq. (11); b) when $\bar{h} > 6.2$, $A = [\overline{Nu}_0\ \bar{R}^2] / \{Pr^{1/3}\bar{h}^{0.63}[1 - (1 - 2.92\bar{R}^2/\bar{h}^{1.4})^{2.2}]\}^{-1}$ [6) when $d_0 = 31$ mm, $\bar{h} = 8–24$, $\bar{R} = 0.83$; 7) 8; 35 and 3.12; 8) 12; 13 and 2.08, authors' tests; 9) 16.5; 8 and 0.5, tests of [8]; 10) 88; 8 and 1.5, tests of [9]; 11) according to Eq. (12)].

To find the heat flux at the wall q_w and the heat transfer coefficient α we use the obvious equality

$$q_w = -\lambda\,(\partial t/\partial y)_{y=0} = \alpha\,(t_w - t_\infty),$$

or

$$\alpha = -\lambda \left(\frac{\partial t}{\partial y}\right)_{y=0} \Big/ (t_w - t_\infty). \tag{5}$$

This requires a knowledge of the temperature distribution in the boundary layer, which, for constant temperature of the whole surface of the disk, we assign in the form of a polynomial of fifth degree

$$t = a + by + cy^2 + dy^3 + ey^4 + fy^5. \tag{6}$$

To find the constants in (6), we use the boundary conditions

$$\text{when } y = 0 \quad t = t_w, \quad \partial^2 t/\partial y^2 = 0;$$
$$\text{when } y = \delta_t \quad t = t_\infty, \quad \partial t/\partial y = 0, \quad \partial^2 t/\partial y^2 = 0, \quad \partial^3 y/\partial y^3 = 0.$$

Using these conditions, (6) may be written in the form

$$t = (t_w - t_\infty)\,(2.5\,y/\delta_t - 5\,y^3/\delta_t^3 + 5\,y^4/\delta_t^4 - 1.5\,y^5/\delta_t^5).$$

We shall evaluate the temperature gradient at the wall, $(\partial t/\partial y)_{y=0}$, and substitute it in (5) to get $\alpha = 2.5\,\lambda/\delta_t$.

In order to interrelate the thicknesses of the thermal and hydrodynamic boundary layers, we shall use the relation derived for longitudinal flow over a uniformly heated plate, $\delta_t/\delta = 1/1.026\ \mathrm{Pr}^{2/3}$.

Thus, the distribution of local heat transfer coefficients over the disk radius in the vicinity of the stagnation point $(0 \le \bar{r} \le \bar{r}_m)$ will be

$$\alpha = \frac{1.3\,\lambda a^{1/2}}{d_0}\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_0^{1/2}\,(1 - k\bar{r}^2)^{1.2},$$

or, in dimensionless form,

$$\mathrm{Nu}_0 = 1.3\,a^{1/2}\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_0^{1/2}\,(1 - k\bar{r}^2)^{1.2}.$$

Substituting the values of a and k in the last equation, we obtain the distribution of local heat transfer coefficients as a function of $\bar{h}$:

$$\text{when } \bar{h} \leqslant 6.2$$
$$\mathrm{Nu}_0 = 1.6\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_0^{1/2}\left(1 - \bar{r}^2/3\bar{h}^{0.2}\right)^{1.2}\bar{h}^{-0.11}, \tag{7}$$

$$\text{when } \bar{h} \geqslant 6.2$$
$$\mathrm{Nu}_0 = 5.25\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_0^{1/2}\left(1 - 2.92\bar{r}^2/\bar{h}^{1.4}\right)^{1.2}\bar{h}^{-0.77}. \tag{8}$$

Letting r = 0 in (7) and (8), we obtain the values of the heat transfer coefficient at the frontal point:

$$\text{when } \bar{h} \leqslant 6.2 \quad \mathrm{Nu}_\kappa = 1.6\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_0^{1/2}\,\bar{h}^{-0.11}, \tag{9}$$

$$\text{when } \bar{h} \geqslant 6.2 \quad \mathrm{Nu}_\kappa = 5.25\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_0^{1/2}\,\bar{h}^{-0.77}. \tag{10}$$

If the disk is entirely located in the region $R \le r_m$, where when $\bar{h} \leqslant 6.2$ $r_m = d_0\,\bar{h}^{0.1}$ and when $\bar{h} \geqslant 6.2$ $r_m = 0.34\,d_0\,\bar{h}^{0.7}$, then the mean heat transfer coefficient over the disk is found from the expression

$$\bar{\alpha} = \frac{1}{\pi R^2}\int_0^R 2\pi\alpha\, r\,dr,$$

or, after integration,

$$\bar{\alpha} = \frac{0.59\,\lambda}{d_0\,a^{-1/2}\,k\bar{R}^2}\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_0^{1/2}\left[1 - \left(1 - k\bar{r}^2\right)^{2.2}\right].$$

Replacing a and k in the last expression by their values, we finally obtain

$$\text{when } \bar{h} \leqslant 6.2 \quad \overline{\mathrm{Nu}}_0 = 2.16\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_0^{1/2} \times$$
$$\times\left[1 - \left(1 - \frac{\bar{R}^2}{3\bar{h}^{0.2}}\right)^{2,2}\right]\bar{h}^{0.09}\,\bar{R}^{-2}, \tag{11}$$

$$\text{when } \bar{h} \geqslant 6.2 \quad \overline{\mathrm{Nu}}_0 = 0.815\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_0^{1/2} \times$$
$$\times\left[1 - \left(1 - \frac{2.92\,\bar{R}^2}{\bar{h}^{1.4}}\right)^{2.2}\right]\bar{h}^{0.63}\,\bar{R}^{-2}. \tag{12}$$

The experimental investigations were carried out on equipment for which a diagram and construction details were given in [2]. In investigating the local coefficients at the stagnation point, use was made of the fact that when $\bar{r} \le 0.5\,\bar{r}_m$, the velocity $\bar{U}_s \approx a\bar{r}$, while the mean heat transfer coefficient deviates negligibly from the local coefficient. Therefore, to determine the local heat transfer coefficients, a compensating heat transfer element, blown upon by nozzles d_0 = 20 and 30 mm in diameter was located at the frontal point at the center of the disk.

The results of the tests are shown in Fig. 1. Also presented are the test data of [6–8], which show satisfactory agreement with the theoretical curves according to (9) and (10). The tests embrace the range of variation Re_0 = 7000–550 000.

Comparison of the theoretical relations (7) and (8) with test data of [6] (Fig. 2) shows that there is good agreement when $\bar{h} \le 6.2$. For larger $\bar{h}$ the divergence of the test data of [6] from the theoretical relation (8) reaches 15%.

The correlation in Fig. 3 spans a wide range of variation of Re_0 = 3800–550 000 and $\bar{h}$ = 1.4–24. The average scatter of the test data does not exceed 15%.

NOTATION

d_0—nozzle diameter; r—variable disk radius; r_m—distance from disk center to point corresponding to maximum value of axial velocity (stagnation point region); R—radius of disk; h—distance from nozzle to heat transfer surface; $\bar{r} = r/d_0$—reduced variable radius; $\bar{R} = R/d_0$—dimensionless disk radius; $\bar{h} = h/d_0$—relative distance from nozzle to disk surface; t_w—temperature of surface; t_∞—temperature outside boundary layer; U_s—velocity at outer edge of boundary layer (axial velocity of jet); U_0—discharge velocity; $\bar{U}_s = U_s/U_0$—dimensionless velocity at outer edge of boundary layer; α—local heat transfer coefficient; $\bar{\alpha}$—mean heat transfer coefficient; $\mathrm{Re}_0 = U_0 d_0/\nu$—Reynolds number; $\mathrm{Nu}_0 = \alpha d_0/\lambda$—local Nusselt number; $\overline{\mathrm{Nu}}_0 = \bar{\alpha} d_0/\lambda$—mean Nusselt number; Nu_K—Nusselt number at stagnation point.

REFERENCES

1. P. M. Brdlik and V. K. Savin, IFZh [Journal of Engineering Physics], **8**, no. 2, 1965.
2. P. M. Brdlik and V. K. Savin, collection: Building Thermophysics [in Russian], Izd. "Energiya," 1966.
3. F. W. Scholkemeyer, Diss. Braunschweig, 1943.
4. A. Walz, Lilienthal-Bericht, 141, 8, 1941.
5. L. G. Loitsyanskii, The Laminar Boundary Layer [in Russian], Fizmatgiz, 1962.
6. R. Gordon and I. Cobongue, Intern. Heat Transfer Conf., II, New York, 1961.
7. H. Jakob, Proc. Phys. Soc., **59**, 1947.
8. K. P. Perry, Proc. Inst. Mech. Eng., 30, 1954.
9. V. N. Timofeev, I. A. Fevraleva, and M. A. Vavilov, Tr. VNII metallurgich. teplotekhniki, no. 8, 1962.

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